# Differentiable Quantum Chemistry

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## Core idea

- Self-consistent cycle can be a pure function
- Build call tree, apply chain rule
- Technical challenges:
  - Requires re-implementation
  - Crossing language borders is hard
  - Some syntactic sugar is unsupported
  - Slow, can be boosted with "Harris functional" like step
- Chemistry challenges:
  - Some derivatives not well defined (e.g. w.r.t. basis set)

## Example Forward Mode

$z = f(x_1, x_2)$	
J (** 1) ** 2)	Оре
$= x_1 x_2 + \sin x_1$	$w_1$ =
$= w_1 w_2 + \sin w_1$	$w_2$ =
$= \eta \eta_{2} + \eta \eta_{4}$	w <sub>3</sub> =
$-\omega_3 + \omega_4$	$w_4$ =
$=w_5$	$w_5$ =

Operations to compute derivative
$\dot{w}_1=1$ (seed)
$\dot{w}_2=0$ (seed)
$\dot{w}_3=w_2\cdot\dot{w}_1+w_1\cdot\dot{w}_2$
$\dot{w}_4 = \cos w_1 \cdot \dot{w}_1$
$\dot{w}_5=\dot{w}_3+\dot{w}_4$

- Most natural
- Compute intensive: needs seed (0, 1) for next variable, so each gradient costs the same as the original function

### Example Reverse Mode

 $z = f(x_1, x_2)$  $= x_1 x_2 + \sin x_1$  $= w_1 w_2 + \sin w_1$  $= w_3 + w_4$ 

 $= w_{5}$ 

- More memory, faster compute
- Multiple derivatives at once
- Can be done on
  - other primitives
  - control structures
- Works with matrices

$$\begin{aligned} \frac{dz}{dz} &= 1\\ z &= w_5 \Rightarrow \frac{dz}{dw_5} = 1\\ \frac{dz}{dw_4} &= \frac{dz}{dw_5} \frac{dw_5}{dw_4} = 1 \times 1 = 1\\ \frac{dz}{dw_3} &= \frac{dz}{dw_5} \frac{dw_5}{dw_3} = 1 \times 1 = 1\\ \frac{dz}{dw_2} &= \frac{dz}{dw_3} \frac{dw_3}{dw_2} = 1 \times w_1 = w_1\\ \frac{dz}{dw_1} &= \frac{dz}{dw_3} \frac{dw_3}{dw_1} + \frac{dz}{dw_4} \frac{dw_4}{dw_1} = w_2 + \cos(w_1) \end{aligned}$$

## DiffiQult

- Forward mode proof-of-concept
- Reason: matrix diagonalisation for degenerate eigenvalues not available in reverse mode
- Optimizes everything
- HF



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**Research Article** 

Automatic Differentiation in Quantum Chemistry with Applications to Fully Variational Hartree–Fock

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#### Quax



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## DQC

- Reverse mode (!)
- Focuses on different properties
- HF, DFT
- Includes alchemical derivatives

Cases	DQC	PySCF
H <sub>2</sub> O (HF/cc-pVDZ)	96 ms	245 ms
H <sub>2</sub> O (PW92/cc-pVDZ)	530 ms	430 ms
C <sub>4</sub> H <sub>5</sub> N (HF/cc-pVTZ)	108 s	17 s
C <sub>4</sub> H <sub>5</sub> N (PW92/cc-pVTZ)	101 s	25 s
C <sub>4</sub> H <sub>5</sub> N (density fit PW92/cc-pVTZ)	30 s	22 s
C <sub>6</sub> H <sub>8</sub> O <sub>6</sub> (density fit PW92/cc-pVDZ)	87 s	57 s

TABLE I. Execution speed comparison between DQC (SCE iterations) and PvSCE



#### DQC: A Python program package for differentiable quantum chemistry

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## PySCF-AD

- Most advanced
- Out-of-core only
- No integral symmetries
- Reverse-Mode
- Takes special care of SCF derivatives
- Based on JAX

