Basis Sets

basissetexchange.org

- Set = multiple basis functions
  - s-orbital: 1 basis function
  - p-orbital: 3 basis functions (px, py, pz)
- Finite
- Big families
  - atom-centered
    - Slater-type orbitals (STO): expensive integrals
    - Gaussian-type orbitals (GTO)
      - Primitive gaussian functions build up orbitals
  - plane waves
  - Wavelets
    - Systematically improving, rare
- Typically defined per element: problem for quantum alchemy
- Connection locality and cost
- Minimal basis set: one basis function for each orbital
- Polarization functions: asymmetry close to the nucleus: bonds
- Diffuse functions: asymmetry away from the nuclei: hydrogen bond, dipoles

$l = 0$

s

$l = 1$

p_x p_z p_y

$l = 2$

$d_{x^2-y^2}$ d_{zx} $d_{3z^2-r^2}$ d_{yx} d_{xy}

$l = 3$ $f_{x(x^2-3y^2)}$ $f_{z(x^2-y^2)}$ $f_{x(5z^2-r^2)}$ $f_{z(5z^2-3r^2)}$ $f_{y(5z^2-r^2)}$ f_{xyz} $f_{y(3x^2-y^2)}$

Plane waves
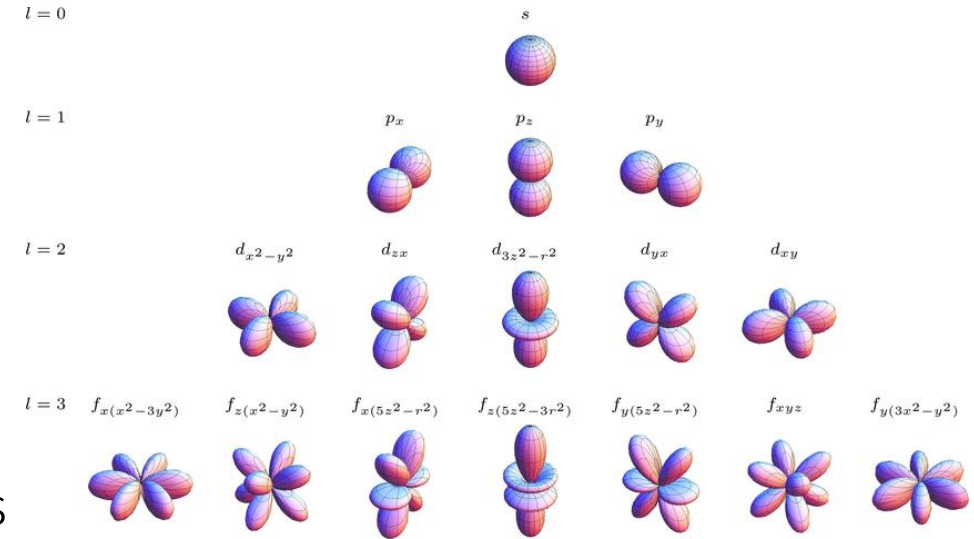
- Path to complete basis set (CBS limit)
- Easy and elegant implementation except HF
- Easy derivatives
- Requires "fake" boundary conditions
- Empty space is not free
- Pseudopotentials required as cutoff too high
- Commonly used for materials

- Localised, so empty space is free
- Scales linearly with number of atoms
- Numerical issues if atoms close together
- Any boundary conditions possible
- No path to complete basis set limit: empirical schemes necessary
- Harder to implement
- Commonly used for molecules / mixed settings

Real-Space
- Multi-dimensional grid
- Easiest to approach CBS
- Hard to implement
- Slow in evaluation
- Mostly used for technical demonstrations / settling one-off questions

Wavelets
- Finite Support: easy to subdivide
- Niche implementations
- Unclear whether "best of both worlds" or "worst of both worlds"

- Gaussian-type orbitals
  - Sum of Gaussians form one orbital: contraction
- Minimal basis set: "single zeta" (Zeta commonly used for the exponent in STO-nG)
- Twice/three times the basis functions for valence atomic orbitals: double-zeta, triple-zeta, …
- Split valence (single zeta for core, multiple for valence)

6-31++G**

6	One core basis function with 6 contracted Gaussians
31	Two valence basis functions with 3 and 1 contracted Gaussian respectively
*	Polarization on heavy atoms
**	Polarization also on hydrogen atoms
+	Diffuse functions on heavy atoms
++	Diffuse function also on hydrogen atoms

Nomenclature

aug-cc-cpCVDZ

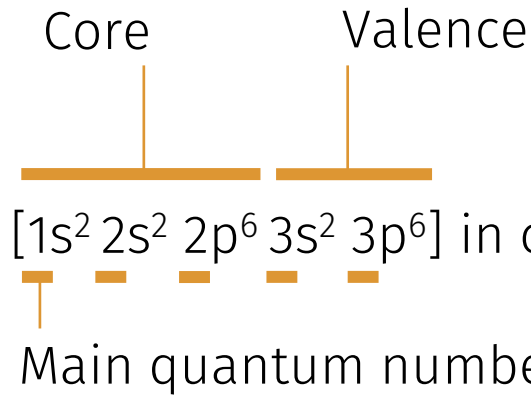|	
aug	Diffuse functions
p	Polarization functions
DZ	Double zeta
C	core basis functions

(6s,3p) → [2s,1p]

Contraction: (primitives) -> [basis functions]

Question: [3s2p1d] is how many basis functions?   (not main quantum numbers)

Example

120

Core   Valence

Ar with electron configuration [$1s^2\,2s^2\,2p^6\,3s^2\,3p^6$] in cc-pVnZ?

Main quantum number

	[all of core]	[valence repeated n times]		[polarization of next higher angular momentum]		
DZ	1s, 2s, 2p	3s, 3s,	3p, 3p	3d	[4s, 3p, 1d]	= 18
TZ	1s, 2s, 2p	3s, 3s, 3s,	3p, 3p, 3p	3d, 4d, 4f,	[5s, 4p, 2d, 1f]	= 34
QZ	1s, 2s, 2p	3s, 3s, 3s, 3s,	3p, 3p, 3p, 3p	3d, 4d, 5d, 4f, 5f, 5g	[6s, 5p, 3d, 2f, 1g]	= 59

…

Minimal basis set (e.g. STO-3g): 1s, 2s, 2p, 3s, 3p   [3s, 2p] = 9

```python
import pyscf.gto as gto
gto.M(atom='Ne 0 0 0', basis='STO-3G').ao_labels()
```

Molecule setup

```
mol_h2o = gto.M(atom = 'O 0 0 0; H 0 1 0; H 0 0 1', basis = 'ccpvdz')

from pyscf import scf
rhf_h2o = scf.RHF(mol_h2o)
e_h2o = rhf_h2o.kernel()
```

Electron density in ao basis

```
dm1_ao = calc.make_rdm1()
```

Electron density on a grid

```
grid = pyscf.dft.gen_grid.Grids(mol)
grid.level = 3
grid.build()
ao_value = pyscf.dft.numint.eval_ao(mol, grid.coords, deriv=0)
rho = pyscf.dft.numint.eval_rho(mol, ao_value, dm1_ao, xctype="LDA")
```