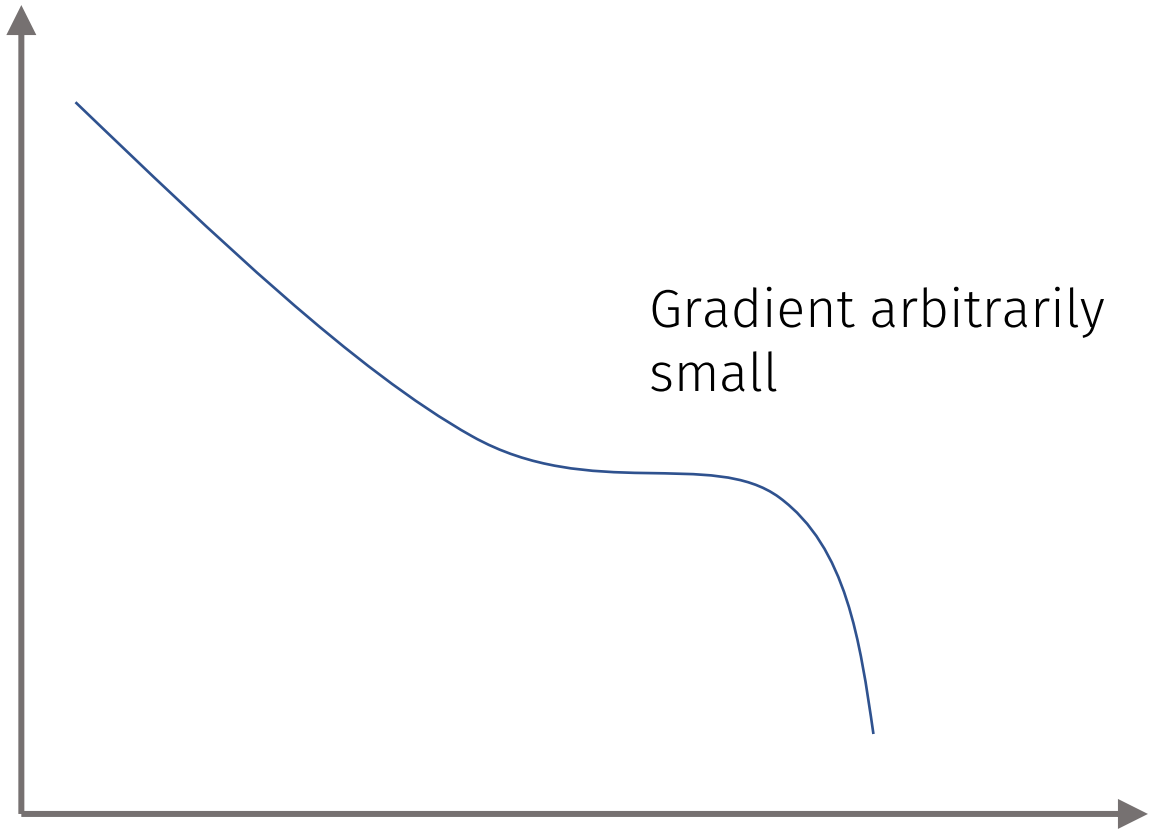
Optimization: Caveats

Convergence
- Hard to establish
- Gradient necessary, but not sufficient
- Hessian expensive
- Local property

Numerical stability
- Finite differences
- Conjugate Gradients
- Shallow minima

Cost of Hessians
- Scales as N^2
  - Water: N=9
  - Caffeine: N=72
- Often only from finite differences



Optimization: Caveats

Curse of dimensionality
- Search space quickly increases
- Often forces tiny optimization steps

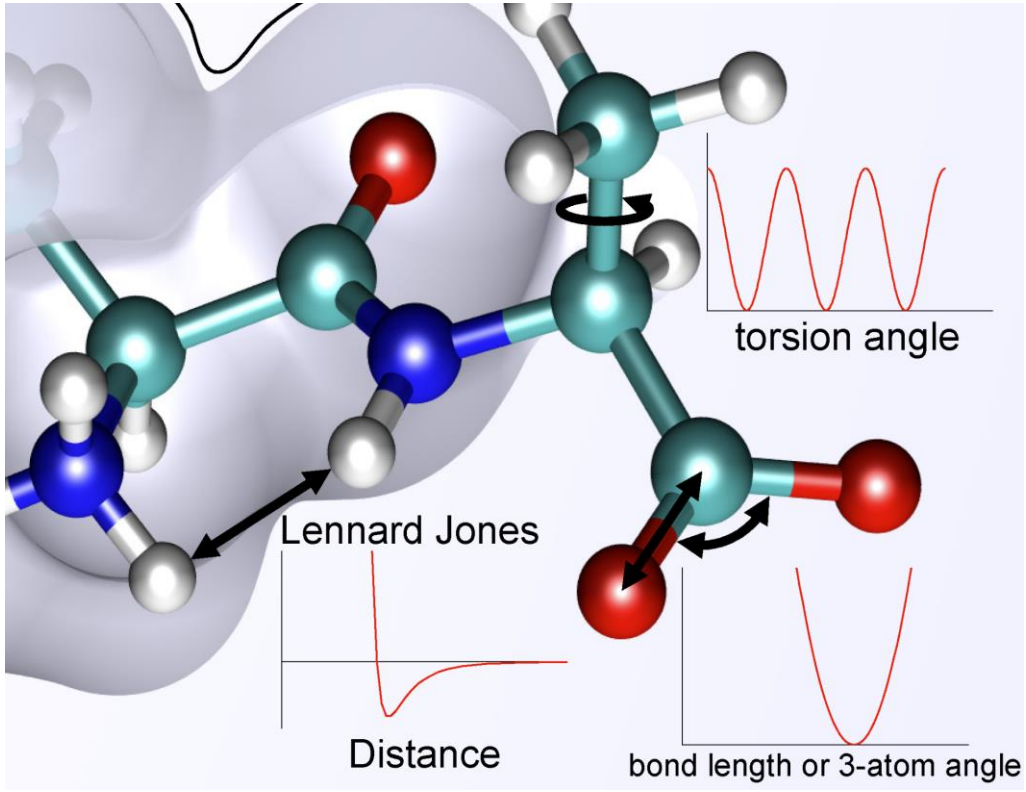
Preconditioning
- Math not equal to finite-precision implementations
- Transform problem into an equivalent one
- Focus on numerical stability
- Key: use libraries when possible or implement algorithms verbatim

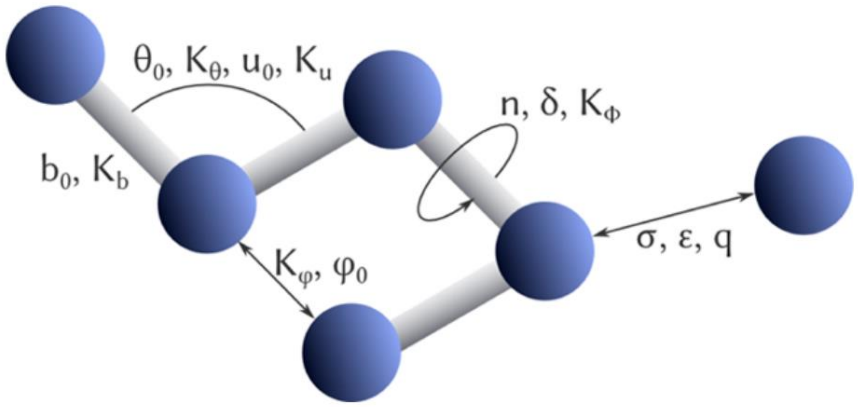
Families of Approximations

Classical molecular dynamics

- Typically fixed bonds
- No quantum effects
- Reference: quantum data

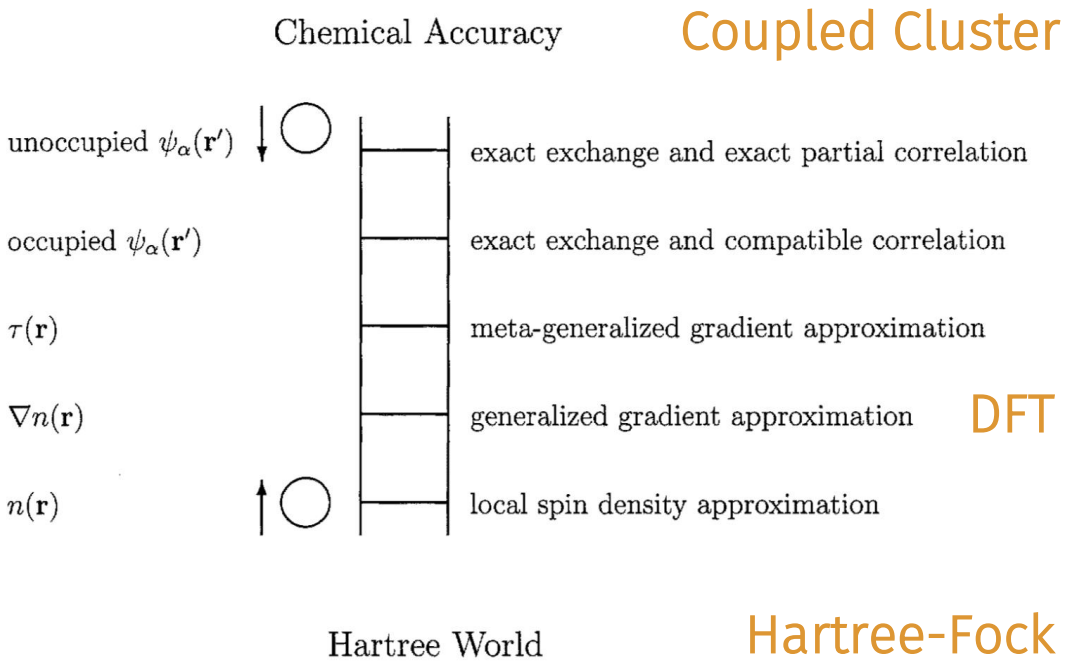
$$E = \sum_{\text{bonds}} K_b(b - b_0)^2 + \sum_{\text{angles}} K_\theta(\theta - \theta_0)^2$$
$$+ \sum_{\text{dihedrals}} K_\phi(1 + \cos(n\phi - \delta))$$
$$+ \sum_{\text{improper}} K_\varphi(\varphi - \varphi_0)^2 + \sum_{\text{Urey-Bradley}} K_u(u - u_0)^2$$
$$+ \sum_{i<j} 4\varepsilon\left[\left(\frac{\sigma_{ij}}{r_{ij}}\right)^{12} - \left(\frac{\sigma_{ij}}{r_{ij}}\right)^{6}\right] + \sum_{i<j} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}}$$





https://dx.doi.org/10.1021/jp507464m / wikicommons

Quantum chemistry



Coupled Cluster

DFT

Hartree-Fock

FIGURE 1. Jacob's ladder of density functional approximations. Any resemblance to the Tower of Babel is purely coincidental. Also shown are angels in the spherical approximation, ascending and descending. Users are free to choose the rungs appropriate to their accuracy requirements and computational resources. However, at present their safety can be guaranteed only on the two lowest rungs.

John Perdew

Michael Willmann, 1691

Machine learning force fields